Defaultable bonds with log-normal spread: an application of the model to Argentinean and Brazilian bonds during the Argentine crisis

MARIANO CANÉ DE ESTRADA mcane@safjp.gov.ar
Superintendencia de AFJPs, Tucumán 500, (1049) Buenos Aires, REPÚBLICA ARGENTINA.

ELSA CORTINA elsa_iam@fibertel.com.ar
Instituto Argentino de Matemática (CONICET), Saavedra 15, 3er. piso, (1083) Buenos Aires, REPÚBLICA ARGENTINA.

CONSTANTINO FERRO FONTÁN ferro@tinfip.lfp.uba.ar
Instituto de Física del Plasma (CONICET), Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria, Pabellón 1, (1428) Buenos Aires, REPÚBLICA ARGENTINA.

JAVIER DI FIORI jdifiori@prorenta.com.ar
Universidad de San Andrés, Vito Dumas 284, (1644) Victoria, Buenos Aires, REPÚBLICA ARGENTINA.

Abstract In this paper we describe a two factor model for a defaultable discount bond, assuming log-normal dynamics with bounded volatility for the instantaneous short rate spread. Under some simplified hipothesis, we obtain an explicit barrier-type solution for zero recovery and constant recovery. We also present a numerical application for Brazilian Sovereign Bonds during the Argentinian default crisis.

Keywords: credit risk, defaultable bonds, log-normal spread.

JEL classification: G 13

Introduction

The approaches to model credit risk can be broadly classified in two classes. The earlier includes the so called structural models, based on the firm's value approach introduced in Merton (1974), and extended in Black and Cox (1976), Longstaff and Schwart (1995) and others.

More recent is the class of the generally termed as reduced-form models, in which the assumptions on a firm's value are dropped, and the default is modeled as an exogenous stochastic process. Reduced-form models have been proposed in Jarrow and Turnbull (1995), Duffie and Kan (1996), Jarrow, Lando and Turnbull (1997), Schonbucher (1998), Cathcart and El Jahel (1998), Duffie and Singleton (1999), Duffie, Pedersen and Singleton (2000), Schonbucher (2000), and others.

A survey of both classes can be seen in Schonbucher (2000), and in Bohn (1999) (models published before 1998). For a detailed overview of reduced-form models published before 1997 see Lando (1997).

The goal in this note is to describe a two factor model where the price of a risky bond price is derived as a function of the risk-free short rate and the instantaneous short spread, and the requirement is that the short spread must be positive. The dynamics of the spread is assumed to satisfy a log-normal diffusion with bounded volatility, and the default occurs if the spread reaches an upper barrier.

Our approach is motivated by a remark in Schonbucher (2000) saying that an alternative to his model of the term structure of defaultable bonds, based on the Heath-Jarrow-Morton (HJM) model (cf. Heath, Jarrow and Morton (1992)), would be a two factor model using an arbitrage free model for the risk-free rate and a model for the forward spread that generates a positive short rate spread.

It is connected to the model presented in Cathcart and El Jahel (1998), since it is also a reduced-form model, solved by a structural approach, that leads to a barrier-type solution; in their model they assume that the default occurs when a signaling process hits some predefined lower barrier.

An extension of Cathcart and El Jahel model is proposed in Lo & Hui (2000), where foreign exchange rates are chosen as the signaling barrier and the dynamics of the default barrier depends of the volatility and drift of the signaling barrier.

Blauer and Wilmott (1998) also use the Black and Scholes option pricing technique to develop a two factor model applied to Brady bonds, but they took expectation on the risk of default instead of hedging it, so our pricing equation and its solution are different from theirs.

The remainder of the paper is organized as follows. in Section 1. we present the model. In section 2 we show the numerical results for an application to Brazilian and Argentinean bonds, and Section 3. contains the conclusions and comments on future work.

1. The model

We work in a continuous time framework, in which $r_d(t)$ is the defaultable short rate if a default event has not ocurred until t, $r(t)$ is the risk-free short rate, and the spread $h(t)$ is defined as

$$h(t) = r_d(t) - r(t).$$

Our assumptions are:

i) at any time t risk-free discount bonds and defaultable discount bonds of all maturities are available,

ii) the dynamics of $r(t)$ and $h(t)$ are governed by diffusion equations

$$dr(t) = \mu_r(r,t)dt + \sigma_r(r,t)dW_1,$$

$$dh(t) = \mu_h(h,t)dt + \sigma_h(h,t)dW_2, \tag{1}$$

where W_1 and W_2 are uncorrelated standard Brownian motions,

iii) the log-volatility σ_h, the drift μ_h, and the market price of default risk λ_h are constant,

iv) the spread is positive, $h(t) > 0$, and the functional form of the volatility in (1) is

$$\sigma_h(h,t) = \min(H_b, h(t))\sigma_h,$$

where H_d is the value of the spread at the date of dafault. In HJM (1992) it is shown that this volatility process gives finite positive rates (spread in this case).

Since r and h were not correlated, the price of a defaultable bond can be written as (c.f. Cortina (2001))

$$P(r,h,t,T) = Z(r,t,T)S(h,t), \tag{2}$$

where $Z(r,t,T)$ is the solution of a risk free bond [1](e.g. Hull & White), and $S(h,t)$ satisfies separates the PDE and leads to

$$\frac{\partial S}{\partial t} + \frac{1}{2}\sigma_h^2(h,t)\frac{\partial^2 S}{\partial h^2} + [\mu_h - \lambda_h\sigma_h]\frac{\partial S}{\partial h} = 0, \tag{3}$$

with the final condition

$$S(h,T) = 1$$

[1]For a full description of interest rate models see Rebonato(1998).

if a default has not occurred until maturity.

For the particular case of a recovery paid in cash, or when it is a fraction of the risk free value, $Q(t) = Q$ is constant; this makes the problem mathematically equivalent to the modeling of a constant rebate for an up-and-out barrier, and the solution to (3) is

$$S(h,t) = Q + (1-Q)\left[N(d_1) - \left(\frac{H_d}{h}\right)^{(k-1)} N(d_2)\right], \quad (4)$$

where H_d is the value of the barrier,

$$d_{1,2}(h,t) = (+,-)\frac{\ln\left(\frac{H_d}{h}\right)}{\sigma\sqrt{(T-t)}} - \frac{1}{2}(k-1)\sigma\sqrt{(T-t)},$$

$N(x)$ is the cumulative probability distribution function for a normally distributed variable with mean zero and variance 1, and k is given as a function of the drift μ_h, the volatility σ_h, and the market price of default risk λ_h, by the espression

$$k = 2\left(\frac{\mu_h}{\sigma_h^2} - \lambda_h \sigma_h\right).$$

2. An application of the Model: Argentina's and Brazil's Sovereign Bonds during the Argentine Crisis.

The market data of the sovereign debt of Argentina and Brazil were fitted into the pricing model developed in previous sections of this paper with the aim of obtaining the implied market expectations over the recovery rate of these bonds and studying their dynamics during the period of unfolding of Argentina's Debt Crisis. To this end, we fed daily market data from Argentinean and Brazilian bonds belonging to JP Morgan's EMBI+ Argentina Index and JP Morgan's EMBI+ Brazil Index, respectively, to the specification of the model given by equation (2).

In equation (4), $h(t)$ is the EMBI+ time series, σ and μ are the log-volatility and drift of the process, and the spread value at default is $H_d = 0.4722$ for Argentina, and was set to $H_d = 0.5$ for Brazil.

Instead of modeling the risk-free term structure we use the present value of risk-free cash flows, and we take as T_i the average life of each bond.

An additional hipothesis, needed for our calculations, is an equal recovery factor Q for all the bonds of the same issuer.

The log-volatilities were estimated by the Exponentially Weighted Moving Average (EWMA) method. Figures 1 and 2 show the behavior of the volatilities for Argentina and Brazil, respectively.

< Insert Figures 1 & 2>

As the model assumes constant volatility, in our calculations we used the following average values

Period	EMBI+AR volatility
05-Sep-01 to 04-Oct-01	0.60
05-Oct-01 to 31-Oct-01	0.48
01-Nov-01 to 30-Nov-01	0.67

Table 1. Average volatilities of EMBI+Argentina

Period	EMBI+BR Volatility
05-Nov-01 to 13-Dec-01	0.42
14-Dec-01 to 10-Jan-02	0.35
11-Jan-02 to 31-Jan-02	0.30
1-Feb-02 to 7-Mar-02	0.24
8-Mar-02 to 17-Apr-02	0.22
18-Apr-02 to 31-May-02	0.27

Table 2. Average volatilities of EMBI+Brazil

In order to completely specify the model, we used sinchronous values of the EMBI+ bonds to estimate cross-sectionally the parameters k and Q, for a series of days, and then examine the time series of parameters produced by the estimation procedure to test whether the empirical results have validated or rejected the model.

Let us consider the sum of the squares of the deviations between model and market bond prices

$$\chi^2 = \sum_{i=1}^{N} n_i \left[B_i - Z_i S_i(Q,k)\right]^2 , \tag{5}$$

where N is number of bonds used in the calculation of the EMBI+, n_i is the weight of bond i in the EMBI+, B_i are the observed daily mid-market bond prices, and Z_i risk-free prices. We look for the set of parameters k and Q that minimize (5). Since one of the parameters appears as argument of an exponential function, the minimization problem is strongly non-linear, and we must search for an adequate local minimum.

Let us recall that k has not an obvious economic meaning, but is only an dimensionless parameter defined for the convenience of the solution of the partial differential equation, while Q, also an adimensional parameter, expresses the recovery as a fraction of the free-risk price.

From the parameters k and Q obtained through minimization we derived the series of daily implied average expected recovery rates R.

In the following three graphics we present the plot of the series of the two mentioned implied parameters for Argentina, and the expected recovery rates coupled with the spread of the EMBI+ Argentina Index, during the period starting on the first days of September 2002 and ending in the last days of November 2002. After this period, the worst of the crisis, the model ceases to provide a good fit to the market data.

< Insert Figure 3>

< Insert Figure 4>

< Insert Figure 5>

To further the evidence from which to extract interesting insights about the level and the dynamics of the implied recovery rates, during the particular timeframe of the crisis, we present below for Brazil the plot of the series of the same two parameters presented in the graphics above, and of expected recovery rates coupled with the spread of the EMBI+ Brazil Index, and during the period starting with November 2002 and ending with the month of May 2003.

< Insert Figure 6>

< Insert Figure 7>

< Insert Figure 8>

The first step to test the validity of the model is to examine the daily series of the estimated parameters. In our model k and Q are not function of time, hence if the model is correct, the estimation procedure should produce the same estimates over the time. In fact, one expects that the parameter were not exactly constant but fluctuating within a statistical noise.

As it is apparent from Figures 6 and 7, the minimization parameters for Brazil lie in a plausible range. k_{BR} oscillates around different constant values, its jumps corresponding to the changes of the average volatility, while Q_{BR} only exhibits a small jump, on January 11. Furthermore, the two average values around which it fluctuates are very close: 0.42 for the first period and 0.39 for the second one.

In Figures 3 and 4 the Argentinean parameters show a similar behavior in the first two periods. However, in the third period, as the EMBI+AR Index increases approaching to the default value, the oscillations of k_{AR} and Q_{AR} become larger. After November 30th. 2001, the lack of stability of the minimization parameters or the inability of the minimization procedure to produce an adequate set of parameters, leaded to the conclusion that the model could no more be validated as a reasonable description of the real process.

Looking at Figures 5 and 8 we find, as expected, a negative significant correlation between the EMBI+ Index for each country and the corresponding expected recovery rate, suggesting that the dynamics of the recovery rate are largely determined by the evolution of the credit spread. In closer examination, we find that the correlation coefficient between these to variables is, in absolute terms, greater in Argentina than in Brazil (-0.96 vs. -0.73). The explanation we found more appropriate for this finding is based in the fact that although the credit spread represents essentially the risk of default, it has also a component that depends on the expectations regarding payoff in case of default. As the probability of default approaches 1, this component becomes more important, given the fact that investors are nearly certain of the upcoming default and the only source of value left is the expected recovery rate after default. We found this explanation also consistent with the fact that the absolute value of the mentioned coefficient of correlation for Argentina is constantly growing as the country approaches its default.

The values observed for R in Figures 5 and 8 provide interesting information about the levels of the expected recovery rate. At the beginning of the dataset, Argentina shows an average expected rate of recovery of approximately 47% of the face value of the bonds. This level is similar to the worst expected recovery rate showed by Brazil (49%) and not very far from Brazil's average of 55% that remains pretty stable during the whole sample. Later as the scenario worsens for Argentina [2], from the last days of October onward, R drops and finally stabilizes around a level of 25%. This level is significantly lower than the evidence for recovery rates from Altman and Eberhart (1994) for US corporate debt (50%), Altman et al. (1999) (40%) and Merrick (1999) for sovereign issues of Argentina during the period of Russia's GKO default crisis in 1998 (50%).

Summarizing the findings of the empirical application of the model, we found that the implied recovery rate level for Brazilian sovereign bonds has persisted during the period of study around 55%, a value not completely out of line with Merrick's findings for Argentina during the Russian crisis. In the case of Argentina the results are quite different, since the model shows very low expected recovery values in comparison with previous cases of default, and very much in line with the first proposals of haircut made by the Argentinean government to the bondholders. Furthermore, the evidence suggests that the major movements observed in the expected recovery rate, were mainly consequence of the credit spread movements to which R is closely linked, especially when the bonds are near default.

5. Conclusions

[2] On October 17th, Standard & Poor's, Moody's Investor Service and Fitch warned that they would rate Argentina in technical default in the case bondholders loose money in a planned domestic swap. Shortly afterwards, Fitch stated that looses to bondholders could total UDS 10 billion. An finally, on November 1st, the president of Argentina and his ministry of economy gave confirmation of the details of a debt swap that resulted in a significant loss of value for the domestic investors participating.

Under simplified assumptions, and modeling the spread as a log-normal random walk with bounded volatility, we have obtained a barrier type closed-form solution for a two factor model of a defaultable discount bond.

This log-normal type model for the spread is the simplest one that satisfies the requirement of positivity, and by relaxing some of the hypothesis it may be improved to better agree with observed phenomenological facts. In particular, in Duffie (1999) it is pointed out that the empirical instantaneous risk of default is mean reverting under the real measure. Therefore, our next step shall be to consider a mean-reverting lognormal type random walk for the spread, and preliminary calculations show that, in this case, a quasi-closed-form solution may be obtained in terms of the confluent hypergeometric functions.

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References

M. Abramowitz and I. Stegun (1970). Handbook of Mathematical Functions. Dover Publications, Inc., New York.

E. I. Altman and A. C. Eberhart (1994). "Do Seniority Provisions Protect Bondholder's Investments?". The Journal of Portfolio Management **20** (4), 67-75.

E. I. Altman, D. Cooke and V. Kishore (1999). Defaults & returns on high yield bonds: analysis through 1998 and default outlook for 1999-2000. New York University Salomon Center.

F. Black and J. Cox (1976). "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions". Journal of Finance **35**(2), 351-367.

J. Bohn (1999). A Survey of Contingent-Claims Approaches to Risky Debt Valuation". Working Paper, Haas School of Business, University of California.

I. Blauer and P. Wilmott (1998). "Risk of Default in Latin American Brady Bonds". Technical report.

L. Cathcart and L. El Jahel (1998). "Valuation of Defaultable Bonds", Journal of Fixed Income, **8**(1), 65-78.

E. Cortina (2001). "A closed-form solution for defaultable bonds with log-normal spread". Publicaciones del instituto Argentino de Matemtica. Nr. 297, pp 15.

D. Duffie and R. Kan (1996). "A Yield-Factor Model of Interest Rates". Mathematical Finance, **6**(4), 379-406.

D. Duffie and K. Singleton (1999). "Modeling Term Structures of Defaultable Bonds". Review of Financial Studies, **12**, 687-729.

D. Duffie, L. Pedersen and K. Singleton (2000). "Modeling Sovereing Yield Spreads: A Case of Study of Russian Debt". Working paper, Graduate School of Business, Stanford University.

B. Goldys, M. Musiela and D. Sondermann (1996). "Lognormality of Rates and Term Structure Models". Working paper No. B-394, University of Bonn.

D. Heath, R. Jarrow and A Morton (1992). "Bond Pricing and the Term Structure of Interest Rates: a New Methodology for Contingent Claims Valuation". Econometrica, **60**(1), 77-105.

M. Hogan (1993). "Problems in Certain Two Factors Term Structure Models". Appl. Prob., **3**, 576-591.

R. Jarrow and S. Turnbull (1995). "Pricing Derivatives on Financial Securities Subject to Credit Risk". Journal of Finance **50**, 53-85.

R. Jarrow, D. Lando and S. Turnbull (1997). "A Markov Model for the Term Structure of Credit Risk Spreads". The Review of Financial Studies **10**, 481-523.

D. Lando (1997). "Modelling Bonds and Derivatives with Default Risk", in Mathematics of Derivatives Securities, ed. by M. Dempster, and S. Pliska. Cambridge University Press, Cambridge, pp. 369-393.

C. F. Lo and C. H. Hui (2000). "Stress testing Model of Defaultable Bond Values". Working paper.

F. Longstaff and E. Schwartz (1995). "A Simple Approach to Value Fixed and Floating Rate Debt". Journal of Finance **50**, 789-819.

J. Merrick. (2000). Crisis Dynamics of Implied Default Recovery Ratios: evidence from Russia and Argentina. Stern School of Business, Ney York University.

R. Merton (1974). "On the Pricing of Corporate Debt: The Risk Structure of Interes Rates". Journal of Finance **29**, 449-470.

K. Miltersen, K. Sandmann and D. Sondermann (1994). "Closed Form Solutions for Term Structure Derivatives with Log-normal Interest Rates". Working paper No. B-308, University of Bonn.

R. Rebonato (1998). Interest Rate Option Models. John Wiley.

D. Sondermann and K. Sandmann (1994). "On the Stability of Log-normal Rate Models". Discussion paper No. B-263, University of Bonn.

P. Schonbucher (1998). "The Term Structure of Defaultable Bonds". The Review of Derivatives Research **2**(2), 161-192.

P. Schonbucher. (2000). "The Pricing of Credit Risk and Credit Risk Derivatives". Discussion paper, University of Bonn.

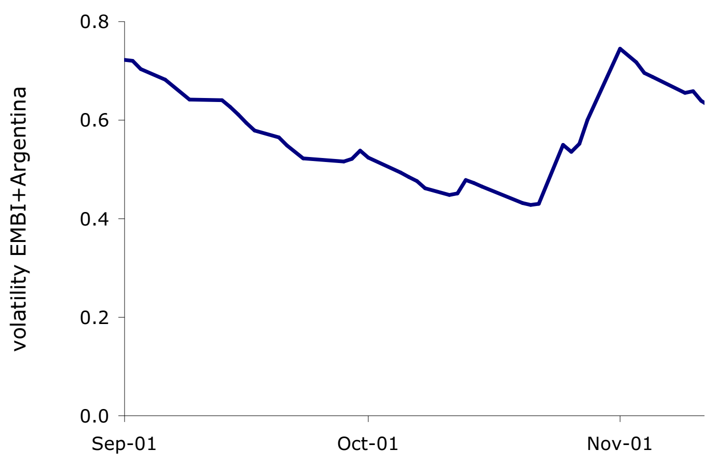

Figure 1: Volatility of EMBI+Argentina index

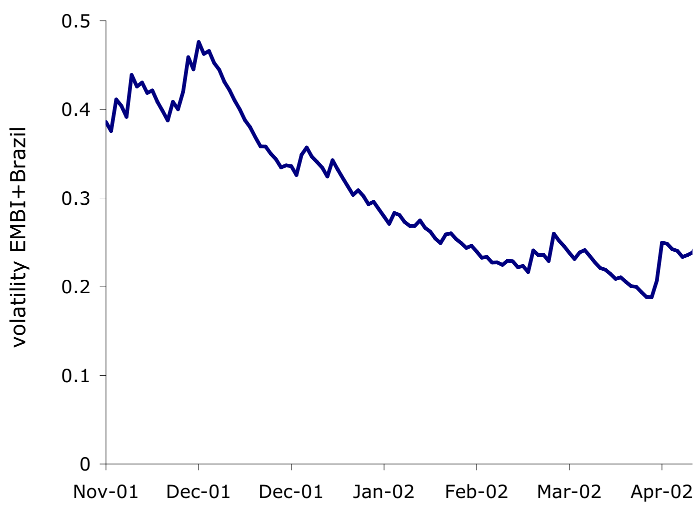

Figure 2: Volatility of EMBI+Brazil index

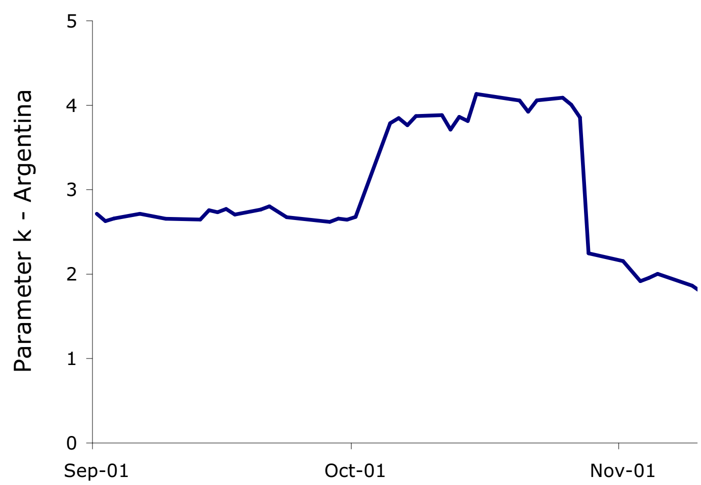

Figure 3: Parameter k for Argentina

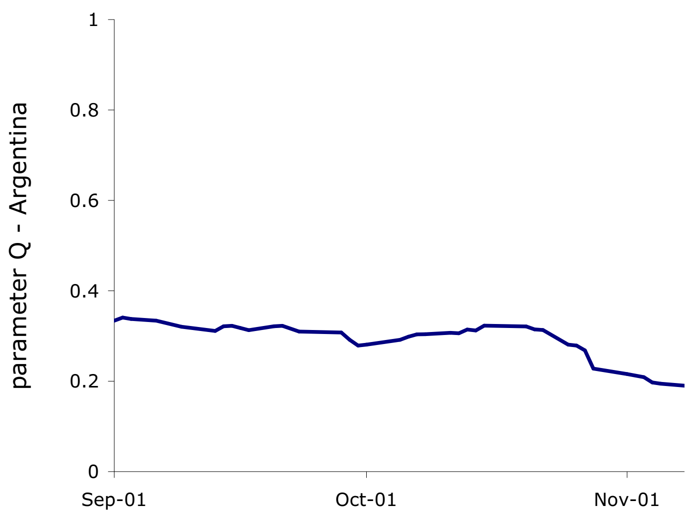

Figure 4: Parameter Q for Argentina

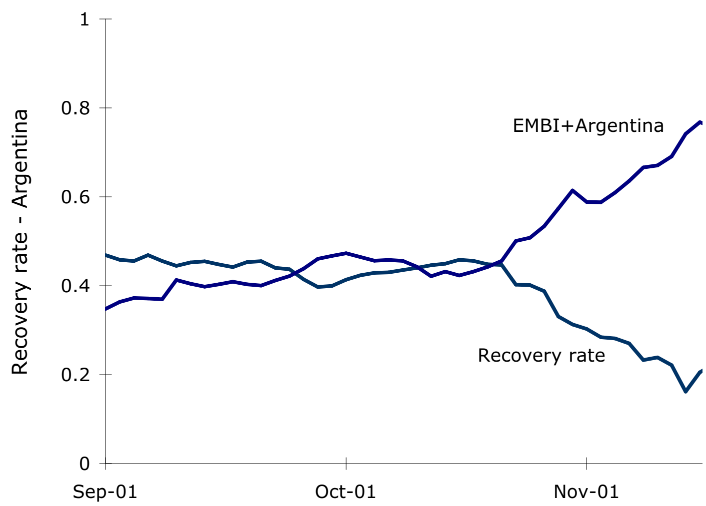

Figure 5: EMBI+Argentina and recovery rate

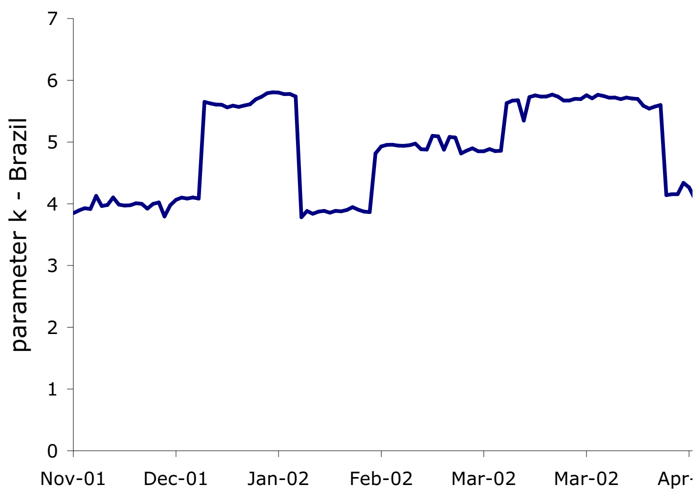

Figure 6: Parameter k for Brazil

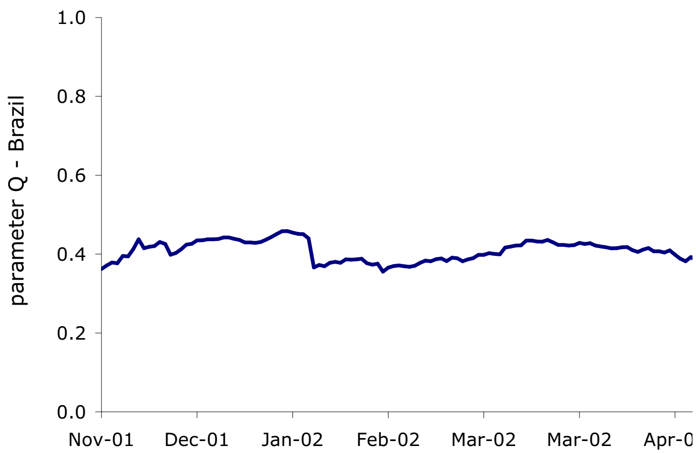

Figure 7: Parameter Q for Brazil

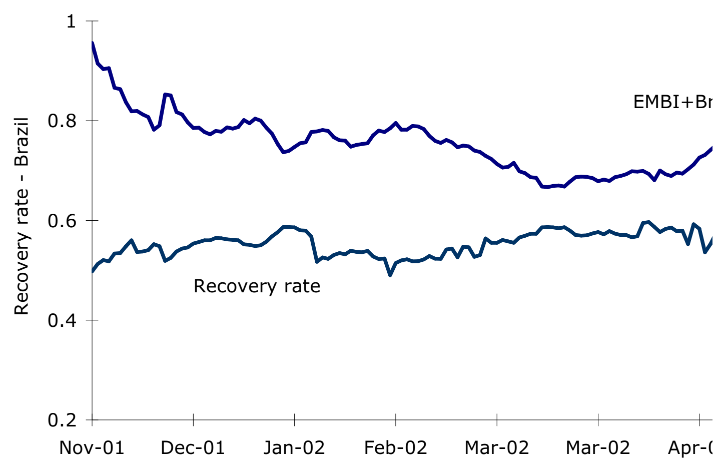

Figure 8: EMBI+Br and recovery rate