THE RESIDUES OF THE RESOLVENT OF THE LAPLACIAN ON DAMEK-RICCI SPACES.

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1. Introduction.

Let N be an H-type group and let S be the canonical solvable extension, as defined in [3]. Harmonic analysis on S has strong similarities with that of symmetric spaces of negative curvature. In particular, it was proved by Damek-Ricci that S is a riemannian harmonic space.

In [7] the resolvent of the Laplacian $R(\mu)$ was studied on symmetric and locally symmetric spaces of negative curvature and in particular it was shown that $R(\mu)$ has a meromorphic continuation to $\mathbf{C}$.

In this paper we announce some recent results on the poles and residues of the resolvent $R(\mu)$ on a general Damek-Ricci space S, namely all poles are simple and the residues are convolution operators which have finite rank (see Theorem 3.4). If S corresponds to a symmetric space of negative curvature, the image of each residue is a $\mathfrak{g}_c$-module with a specific highest weight, hence its dimension can be computed by Weyl dimension formula (see Theorem 3.5). We specialize our results at the end, by discussing the case of the real hyperbolic n-space, which was treated by Guillopé-Zworski in [4].

2. Preliminaries.

In this section we will recall some basic notions on H-type groups and their canonical solvable extensions, following mainly [3] (see also [1]). We will also review some basic facts on symmetric spaces of negative curvature.

Let $\mathfrak{n}$ be a two-step real nilpotent Lie algebra endowed with an inner product $\langle\ ,\ \rangle$. Assume $\mathfrak{n}$ has an orthogonal decomposition $\mathfrak{n}=\mathfrak{z}\oplus\mathfrak{v}$, where $\mathfrak{z}$ is the center of $\mathfrak{n}$ and $[\mathfrak{v},\mathfrak{v}]=\mathfrak{z}$. Define a linear mapping $J:\mathfrak{z}\to\mathrm{End}(\mathfrak{v})$ by

$$\langle J_Z X, Y\rangle = \langle Z,[X,Y]\rangle \tag{1}$$

(note that J_Z is skew-symmetric). Now $\mathfrak{n}$ is said to be an *H-type algebra* if for any $Z_1, Z_2\in\mathfrak{z}$

$$J_{Z_1}J_{Z_2}+J_{Z_2}J_{Z_1} = -2\langle Z_1,Z_2\rangle \tag{2}$$

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The corresponding H-type group is the simply connected Lie group N with Lie algebra $\mathfrak{n}$ endowed with the left invariant metric induced by the inner product $\langle\ ,\ \rangle$ on $\mathfrak{n}$.

Consider the solvable extension, S, of N, as defined in [3], that is, $A = R^+$ acts on N by the dilation $(x, z) \to (t^{\frac{1}{2}}x, tz)$, and $S = AN$ is the semidirect product of A and N. If $\mathfrak{s}$, $\mathfrak{a}$, $\mathfrak{n}$ denote respectively the Lie algebras of S, A and N, we have that $\mathfrak{s} = \mathfrak{a} \oplus \mathfrak{n}$, where $\mathfrak{a} = \mathbf{R}H$ acts on $\mathfrak{n}$ via H, the derivation of $\mathfrak{n}$ such that $H|_{\mathfrak{v}} = \frac{1}{2}I$ and $H|_{\mathfrak{z}} = I$. We use the inner product on $\mathfrak{s}$ extending the one on $\mathfrak{n}$ and such that $\|H\| = 1$, $\langle H, \mathfrak{n}\rangle = 0$; S carries the induced left invariant riemannian structure. Also, we shall denote $q = \dim \mathfrak{z}$, $p = \dim \mathfrak{v}$, $n = \dim \mathfrak{s} = p + q + 1$ and $Q = \frac{1}{2}(p + 2q)$. We point out that if $\mathfrak{n}$ is abelian we shall use the convention that $\mathfrak{v} = 0$, and $\mathfrak{n} = \mathfrak{z}$.

Using coordinates form $\mathfrak{v} \oplus \mathfrak{z} \oplus \mathbf{R}^+$, the product on S is expressed as

$$(Z, Z, a)(X', Z', a') = (X + a^{\frac{1}{2}}X', Z + aZ' + \frac{1}{2}a^{\frac{1}{2}}[X, X'], aa')\ .$$

Note that the volume element of the induced left invariant riemannian metric g on S is the left Haar measure

$$dm = a^{-Q-1}dXdZda\ .$$

We will need the fact that S can be realized as the unit ball in $\mathfrak{s}$:

$$B(\mathfrak{s}) = \{(X, S, u) : |X|^2 + |Z|^2 + u^2 = 1\}$$

via a Cayley type transform $\tilde{C} : S \to B(\mathfrak{s})$, where $\tilde{C} = C^{-1}h$ in the notation of [3], Section 4. In $B(\mathfrak{s})$ the geodesics through the origin are the diameters and the geodesic distance to the origin is given by $r = d(\tilde{p}, 0) = \log \frac{1+|\tilde{p}|}{1-|\tilde{p}|}$, thus $|\tilde{p}| = \tanh(r/2)$, where $\tilde{p} = \tilde{C}(p)$ if $p \in S$. Since

$$|\tilde{C}^{-1}(X, Z, a)|^2 = 1 - \frac{4a}{(1 + a + \frac{1}{4}|X|^2)^2 + |Z|^2}$$

we have that

$$\cosh(r/2)^{-2} = \frac{4a}{(1 + a + \frac{1}{4}|X|^2)^2 + |Z|^2} \tag{3}$$

and furthermore, the image of the left Haar measure on S via $\tilde{C}^{-1}$ is $d\mu = J(r)d\sigma dr$ where r, σ are the radial coordinates on B, $r^2 = |X|^2 + |Z|^2 + u^2$ and $J(r) = 2^p\ \sinh(r/2)^p\ \sinh(r)^q$ (see [3] Section 4).

The symmetric spaces of negative curvature are a main subclass of the Damek-Ricci spaces. Let G be a connected, non compact, semisimple Lie group of real rank one. Let K be a maximal compact subgroup of G and let $\mathfrak{g}$ and $\mathfrak{k}$ be the corresponding Lie algebras. If $G = NAK$ is an Iwasawa decomposition of G, then N is an H-type group, and $S = NA \approx G/K$ is a solvable group in the class introduced above. Indeed, if $\mathfrak{a}$ and $\mathfrak{n}$ denote the Lie algebras of A and N respectively, then $\mathfrak{n}$ splits $\mathfrak{n} = \mathfrak{g}_{\alpha/2} \oplus \mathfrak{g}_{\alpha}$, where $\mathfrak{g}_{j\alpha}$, $j = 1/2, 1,$ denote the $j\alpha$-root spaces of $\mathfrak{a}$. In the notation above we have $\mathfrak{n}_{\alpha} = \mathfrak{z}$, $\mathfrak{n}_{\alpha/2} = \mathfrak{v}$, and $H = adH_o$ with $H_o \in \mathfrak{a}$ such that $\alpha(H_o) = 1$. Then $Q = 2\rho$, where $\rho = \frac{1}{4}(p + 2q)$. If on $S = NA$ we use the G-invariant metric induced by $2(p+4q)^{-1}B$ (B the Killing form of $\mathfrak{g}$) then S is isometric to a Damek-Ricci space. We note that, because of our convention, if $\mathfrak{n}$ is abelian then $p = 0$, $q = \dim \mathfrak{n}$.

3. The residues of the resolvent of the Laplacian on S.

Damek-Ricci spaces have similarities with symmetric spaces of negative curvature, in particular they are harmonic spaces. On S there is a radialization operator π which corresponds to the standard operator in the case of the ball model of S (see [3] p. 230). If $f \in C_c^{\infty}(S)$, $p \in S$ and $\tilde{p} = \tilde{C}(p)$ then

$$\pi f(p) := \int_{S^{p+q}} \tilde{f}(\|\tilde{p}\|\sigma) d\sigma$$

where $\tilde{f} := f \circ \tilde{C}^{-1}$. This operator corresponds in the symmetric case to averaging over the action of K by left translation: if $f \in C^{\infty}(NA)$, then

$$\pi f(x) = \int_K \tilde{f}(kx) dk \tag{4}$$

where $\tilde{f}$ denotes the right K-invariant extension of f.

If $\{Z_i\}$, $\{V_j\}$ are orthonormal bases of $\mathfrak{z}$ and $\mathfrak{v}$ respectively, the Laplacian-Beltrami operator is given by $L = \sum_i Z_i^2 + \sum_j V_j^2 + H^2 - QH$ (see [2]). Since L is self-adjoint and S is a harmonic manifold, then L commutes with π. Moreover, L generates the algebra of left-invariant differential operators on S which commute with π (see [3] Theorem 5.2).

If f is smooth radial function on $S - \{e\}$, we will often abuse notation by writing $f(r) = f(x)$, where $r = d(x, e)$. The action of L on radial functions is thus given by (see [3])

$$Lf(r) = \frac{d^2}{dr^2} f(r) + \frac{1}{2}(p \coth(r/2) + 2q \coth(r)) \frac{d}{dr} f(r) . \tag{5}$$

In the symmetric case, if $\mathfrak{n}$ is not abelian and we set $r = 2t$, then $Lf(t)$ corresponds to $\frac{1}{4} Cf(a_t)$,in the notation of [7] Section 1 (1). If $\mathfrak{n}$ is abelian, L corresponds to C.

A *spherical function* ψ on S is a radial eigenfunction of the Laplace Beltrami operator such that $\psi(e) = 1$. This generalizes the corresponding notion in the symmetric case and has the following characterization ([3]):

Proposition 3.1. *Let $\nu \in \boldsymbol{C}$. The function $\phi_\nu = \pi(a^{\nu+Q/2})$ is a spherical function with eigenvalue $\lambda(\nu) = \nu^2 - Q^2/4$ and any spherical function on S is of this form. Furthermore $\phi_\nu = \phi_{-\nu}$, for each $\nu \in \boldsymbol{C}$.*

As in the symmetric case, we can express ϕ_ν by a hypergeometric function as follows. By letting $z = -\sinh(r/2)$ equation

$$\left\{\frac{d^2}{dr^2} + \frac{1}{2}(p\coth(r/2) + 2q\coth(r))\frac{d}{dr} - \lambda(\nu)\right\} f_\nu(r) = 0 \tag{6}$$

transforms into the hypergeometric equation with parameters $a = Q/2 - \nu$, $b = Q/2 + \nu$, and $c = n/2$. Since $\phi_\nu(e) = 1$, it follows that

$$\phi_\nu(r) = F\left(-\nu + Q/2, \nu + Q/2, \frac{n}{2}, -\sinh(r/2)^2\right) \tag{7}$$

where $F(a, b, c, z)$ denotes the hypergeometric function. Furthermore, if $\operatorname{Re}\nu > 0$, the asymptotic behavior of $\phi_\nu(r)$, as $r \to \infty$, is given by (see [3], p. 239)

$$\phi_\nu(r) \sim c(\nu) e^{r(\nu+Q/2}, \quad \text{where} \quad c(\nu) = \frac{2^{-2\nu+Q}\Gamma(n/2)\Gamma(2\nu)}{\Gamma(\nu + Q/2)\Gamma(\nu + \frac{p+2}{4})}\,. \tag{8}$$

Here $c(\nu)$ coincides with Harish Chandra's c-function in the symmetric case. The Plancherel measure, $\mu(\nu) = (c(\nu)c(-\nu))^{-1}$, can be written $\mu(\nu) = c_o p(\nu) D(\nu)$, where $p(\nu)$ is the polynomial given by:

$$\begin{array}{ll}
\displaystyle\prod_{j=0}^{\frac{p}{4}-1}\left(-\nu^2 + \left(\frac{2j+1}{2}\right)^2\right)\prod_{j=0}^{\frac{Q}{2}-1}\left(-\nu^2 + \left(\frac{j}{2}\right)^2\right), & q,\ \frac{p}{2}\ \text{even}. \\
\displaystyle -\prod_{j=1}^{p/4}(-\nu^2 + j^2)^2\nu^3\,, & q = 1,\ \frac{p}{2}\ \text{odd}. \\
\displaystyle -\prod_{j=0}^{\frac{p}{4}-1}\left(-\nu^2 + \left(\frac{2j+1}{2}\right)^2\right)\prod_{j=0}^{\frac{Q}{2}-1}\left(-\nu^2 + \left(\frac{2j+1}{2}\right)^2\right)\nu\,, & q\ \text{odd},\ \frac{p}{2}\ \text{even}.
\end{array}$$

c_o is a constant, and in each case $D(\nu)$ is given respectively by 1, $\cot(\pi\nu)$, and $\operatorname{tg}(\pi\nu)$ ([ADY]).

Remark 3.2. *We note that* p *es always even, since* $\mathfrak{v}$ *is a module over the Clifford algebra of* $\mathfrak{z}$ *. If* $p=0$ *, then* $X \approx H^{q+1}$ *,* $G \simeq SO(q+1,1)$ *and in this case* $D(\nu)$ *equals 1 or* $\mathrm{tg}(\pi\nu)$ *depending on whether* q *is even or odd.*

In [7], the resolvent of the Laplacian was studied on symmetric (and locally symmetric spaces) of negative curvature. In the symmetric case it was shown that it is given by convolution with a smooth radial function Q_ν on $S-\{e\}$ which is an eigenfunction of L with eigenvalue $\lambda(\nu)=\nu^2-\frac{Q^2}{4}$. It was also shown that $R(\lambda(\nu))$ has a meromorphic continuation to $\mathbf{C}$. These properties remain valid for any S as above and some arguments in [7], can be adapted. On the other hand, it is more convenient to give a different construction of Q_ν, using a series solution.

Now for $b \in \mathbf{R}$ and $\delta > 0$, let $\mathcal{S}_{b,\delta} = \{\nu : \mathrm{Re}\,\nu > b$ and $|\nu+j| > \delta$ $\forall j \in -\mathbf{N} : b \leq j\}$. That is $\mathcal{S}_{b,\delta} = \{\nu : \mathrm{Re}\,\nu > b\}$, if $b \geq 0$, and $\mathcal{S}_{b,\delta}$ is a half plane with finitely many discs removed, centered at $-1,-2,\ldots,-k$, where $-k \geq b$, if $b<0$. The following theorem gives the main properties of Q_ν.

Theorem 3.3. *If* $\nu \in \boldsymbol{C}$, $2\nu \notin -\boldsymbol{N}$ *, then there exists a radial function* $Q_\nu \in C^\infty(S-\{e\})$ *with the following properties*

(a) $(L-\lambda(\nu))Q_\nu = 0$ *and, for each* $s \in S$ *the map* $\nu \mapsto Q_\nu(s)$ *is holomorphic for* $2\nu \notin -\boldsymbol{N}$ *. Furthermore, where defined,*

$$\phi_\nu = c(-\nu)Q_\nu + c(\nu)Q_{-\nu} \tag{9}$$

(b) *If* $f \in C^\infty(S-\{e\})$ *is radial and* $Lf=\lambda(\nu)f$ *with* $2\nu \notin -\boldsymbol{N}$ *then* $f = a\phi_\nu + bQ_\nu$ *, for some* $a,b \in \boldsymbol{C}$ *.*

(c) *There exists a meromorphic function* $d(\nu)$ *on* $\boldsymbol{C}$ *, holomorphic if* $2\nu \notin -\boldsymbol{N}$ *, such that, as* $r \mapsto 0$ *,*

$$Q_\nu(r) \sim d(\nu) r^{p+q-1} |\log r|^{\delta_{p+q,1}} \ .$$

(d) $\lim_{r\to 0^+} J(r)\frac{d}{dt}Q_\nu(r) = -2\nu c(\nu)$.

(e) *If* $f \in C_c^\infty(S)$ *and* $2\nu \notin -\boldsymbol{N}$ *then*

$$\int_S Q_\nu(x^{-1}y)(L-\lambda(\nu)I)f(y)dy = -2\nu c(\nu) f(x) \ . \tag{10}$$

(f) *For any* $b \in \boldsymbol{R}$ $\delta, r_o > 0$ *, there exists a constant* $K = K(b,\delta,r_o)$ *such that* $|Q_\nu(r)| \leq K$ *for any* $r \geq r_o$ *,* $\nu \in \mathcal{S}_{b,\delta}$ *.*

Let $R(\mu)$ denote the resolvent of L at μ. By (10), if $\lambda(\nu) = \nu^2 - \frac{Q^2}{4}$, then $R(\lambda(\nu))$ is given by a kernel operator with kernel $K_\nu(x,y) = -\frac{Q_\nu(x^{-1}y)}{2\nu c(\nu)}$. The following result gives the poles and residues of $R(\lambda(\nu))$, therefore the poles of $\nu \mapsto -\frac{Q_\nu(x^{-1}y)}{2\nu c(\nu)}$.

Theorem 3.4. *If p,q are both even then $R(\lambda(\nu))$ is everywhere holomorphic. Otherwise, it has simple poles lying at $\nu_k = -Q/2 - k$ with $k \in \mathbf{N} \cup \{0\}$. If $\nu = \nu_k$, then $\mathrm{Res}_{\nu=\nu_k} R(\lambda(\nu)) = (2\pi\nu_k)^{-1} p(\nu_k) f * \phi_\nu$ is a finite rank operator, for each k. Here $p(\nu)$ is the polynomial part of the Plancherel measure $\mu(\nu)$.*

In the case when S is of symmetric type one can get more precise information on the operators T_{ν_k}, by using representation theory. To do this we will first introduce some notation. The group of isometries G of S is a noncompact semisimple Lie group of real rank one.

Let $\mathfrak{g}$, $\mathfrak{k}$, N, and A be as in Section 1, let M be the centralizer of A in K, $P = MAN$ and let $\mathfrak{p}$ be the Lie algebra of P. Extend $\mathfrak{a}$ in the usual way to a Cartan subalgebra $\mathfrak{h}_c = \mathfrak{a}_c + \mathfrak{h}_c^-$ of $\mathfrak{g}$, where $\mathfrak{h}^-$ is a maximal abelian subalgebra of $\mathfrak{m}$, and introduce compatible orderings in the dual spaces of $\mathfrak{a}$ and $\mathfrak{a} + \sqrt{-1}\mathfrak{h}^-$.

Let $\Sigma^+(\Delta^+)$ denote the corresponding set of positive roots of the pair $(\mathfrak{g}, \mathfrak{a})$ (respectively $(\mathfrak{g}_c, \mathfrak{h}_c)$).

For $\nu \in \mathbf{C}$, let (π_ν, H^ν) be the spherical principal series representation of G (see [8], Section 3.6). The zonal spherical function ϕ_ν is given by

$$\phi_\nu(g) = \langle \pi_\nu(g) 1_\nu, 1_\nu \rangle \tag{11}$$

where $1_\nu \in H^\nu$ is the K-fixed vector given by $1_\nu(nak) = a^{(\nu+\rho)\alpha}$, $n \in N$, $a \in A$, $k \in K$, and $\langle \, , \, \rangle$ is the standard inner product on H^ν.

In the simmetric case, one can obtain more information on the dimension of $\mathrm{Res}_{\nu=\nu_k} R(\lambda(\nu)) = (2\pi\nu_k)^{-1} p(\nu_k) f * \tilde{\phi}_\nu$.

Theorem 3.5. *Let $S = G/K$ be a noncompact symmetric space of real rank one and let $\nu_k = -\rho - k$ with $k \in \mathbf{N} \cup \{0\}$. Then $Im(Res_{\nu=\nu_k} R(\lambda(\nu))$ is an irreducible $\mathfrak{g}_c$-module of highest weight $k\alpha$. In particular, its dimension is given by the Weyl dimension formula.*

Example 3.6. *The real hyperbolic n-space.*

In [4], Section 2, Guillopé-Zworski consider the resolvent in the case when $S \approx H^n$, finding its poles and showing that the residues are operators of finite-rank. In this particular case one can obtain an explicit series expression for Q_ν:

$$Q_\nu(r) = e^{-(\nu+\rho)r} \sum_{j=0}^{\infty} c_j(\nu) e^{-2jr}$$

and for the residues of $R(\lambda(\nu))$. If we choose $c_o = 1$, we have for $j \geq 1$:

$$c_j(\nu) = \frac{(\rho)_j(\nu+\rho)_j}{j!\ (\nu+1)_j}\ . \tag{12}$$

Furthermore, $c(\nu)$ can be written:

$$c(\nu) = \frac{2^{2\rho-1}\Gamma(n/2)}{\pi^{1/2}}\frac{\Gamma(\nu)}{\Gamma(\nu+\rho)} \tag{13}$$

and now, using the duplication formula for the Gamma function, we obtain:

$$\frac{Q_\nu(r)}{2\nu c(\nu)} = \frac{2^{-2n+3}}{(n-2)!}e^{-(\nu+\rho)r}\sum_{j=0}^{\infty}\frac{\Gamma(\rho+j)}{j!}\frac{\Gamma(\nu+\rho+j)}{\Gamma(\nu+j+1)}e^{-2jr}\ . \tag{14}$$

Also, if we take $\mathcal{S}_{b,\delta}$ as in Theorem (3.3), it is not hard to see, by using Stirling's estimates, that there exists a constant $K = K(b,\delta)$ such that the coefficients in the series are bounded by $Kj^{\rho-1}|\nu+j|^{\rho-1}$, uniformly for ν in $\mathcal{S}_{b,\delta}$.

If n is odd, $\rho = \frac{n-1}{2} \in \mathbf{N}$, hence the coefficients of the series in (14) are polynomial functions in ν and $R(\lambda(\nu))$ is everywhere holomorphic, in this case.

On the other hand, if n is even, (14) implies that the kernel of $R(\lambda(\nu))$ is a meromorphic function with poles at $\nu_k = -\rho-k$, $k \in \mathbf{N}\cup\{0\}$.

Since $\Gamma(\nu+\rho+j)$ is holomorphic at $\nu = \nu_k$ for $j > k$, in this case the residues are

$$\begin{aligned}\mathrm{Res}_{\,\nu=\nu_k}\tfrac{Q_\nu(r)}{2\nu c(\nu)} &= \tfrac{2^{-2n+3}}{(n-2)!}\sum_{j=0}^{k}\frac{\Gamma(\rho+j)(-1)^{k-j}}{j!(k-j)!\Gamma(-k-\rho+j+1)}e^{-(2j-k)r}\\ &= \tfrac{2^{-2n+4}}{(n-2)!}\sum_{j=0}^{[n/2]}\frac{\Gamma(\rho+j)(-1)^{k-j}}{j!(k-j)!\Gamma(-k-\rho+j+1)}\cosh(2j-k)r\end{aligned}$$

since $\frac{\Gamma(\rho+j)(-1)^k}{\Gamma(-k-\rho+j+1)} = \frac{\Gamma(\rho+k-j)}{\Gamma(-\rho-j+1)}$, for $0 \leq j \leq k$.

We note finally that in this case $G = SO(n,1)$, $K = SO(n-1)$ and one finds that the highest weight of the representation in Theorem 3.5 is $k\lambda_1$ where λ_1 is the first fundamental weight. Thus, V_k is isomorphic to the representation of G on $\mathcal{H}_k$, the space of homogeneous harmonic polynomials of degree k and the image of $\mathrm{Res}_{\,\nu=\nu_k}R(\lambda(\nu))$ has dimension $\frac{(k+n-3)!(2k+n-2)}{k!(n-2)!}$ (see [6] Ch.4, (4.12)).

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